GLAUBER – SITENKO SCREENING IN ELASTIC AND INELASTIC DIFFRACTION OF HADRONS AND LIGHT NUCLEI

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The basic reason of usages theory of repeated diffraction scattering for analysis of nuclear-nuclear scattering serves it success in description of processes of hadron-nucleus interactions. At the nuclear-nuclear scattering theoretical analysis we use the same approximations, that at hadron-nucleon scattering [1-15].

In the basic of hadron-nuclear scattering theory in the eikonal approximations there are three physical assumptions: first of all, that during the time of flight of relativistic particle through nucleus the nucleon does not change its position inside the nucleus, which means that we have scattering on the system of fixed scattering centers; secondly, we suggest that the phase shift of impact parameter in the space takes place, thirdly, usage of the completeness of wave functions of target nucleus final states [1-8].

Using previous notations in [1-3, 7, 8, 15], results of scattering cross-section of A1 nucleus on the target nucleus A2 can be present as:

\[
\frac{d\sigma}{d\Omega}^{q.el} = K(q) \left( \frac{p}{2\pi} \right)^2 \int db_1 db_2 \exp \left( i q \left( b_1 - b_2 \right) \cdot \sum_{k=1}^{d_1} \rho_{\delta_k} (q, \vec{r}_k) dr_k \right) \cdot \\
\prod_{j=1}^{d_2} \rho_{\delta_j} (r) dr_i \cdot \exp \left[ \int \left[ \omega \left( b_1 - \vec{s} ; \{s\} \right) + \omega^* \left( b_2 - \vec{s} ; \{s\} \right) \right] dr_1 (\vec{s}) d\vec{s} \right] \\
\cdot \left\{ \exp \left[ \int \omega \left( b_1 - \vec{s} ; \{s\} \right) \omega^* \left( b_2 - \vec{s} ; \{s\} \right) dr_1 (\vec{s}) d\vec{s} \right] - 1 \right\}
\]

For calculation of cross-sections of quasielastic scattering it is necessary to elaborate approximated procedure calculation. One of probable possibilities consists of simplifications in the decomposition values in series, on so-called multiplicities of quasielastic scattering, mathematically corresponding to the exponent expansion in series in curly brackets of expressions (1).
\[
\left( \frac{d\sigma}{d\Omega} \right)_{q,el} = \sum_{n=1}^{\infty} \left( \frac{d\sigma}{d\Omega} \right)_{n},
\]

where

\[
\left( \frac{d\sigma}{d\Omega} \right)_{n} = \frac{[K(q)]^{2}}{n!} \int \prod_{i=1}^{n} T(\tilde{s}, \tilde{s}) d^{2}\tilde{s} |F^{(n)}(q, \{s\})|^{2},
\]

\[
F^{(n)} = \frac{i p}{2\pi} \int db \exp i\tilde{q} \tilde{b} \sum_{k=1}^{n} \rho_{\alpha}(r_{k}) d^{3}r_{k} \cdot \prod_{i=1}^{n} \omega(b - \tilde{s}, \{s\}) \exp \left[ - \int \omega(b - \tilde{s}, \{s\}) \cdot T(\tilde{s}) d^{2}\tilde{s} \right] \]

Such procedure is possible, if in the sum (2) the main contribution comes from the low order terms (n=1,2,3,..). As it follows from the analysis of proton-nuclear quasielastic scattering of [7,8], this condition is valid only at small values of transferred momentum. Relative role of the n-multiplicities of quasielastic scattering, corresponding to backing-out from target nucleus of n-nucleons is determined by the factor [12]

\[
\varepsilon^{-1} \exp \frac{n-1}{n} B q^{2}, \quad \varepsilon = \frac{\sigma_{el}}{\sigma_{t}} = \frac{\sigma_{t}}{16\pi B}
\]

where \(\sigma_{t}\) - total cross section of NN-interactions, \(B\) - slope parameter of elastic NN-scattering.

From general reasons leads that for the qualitative estimations of the contributions of different terms in the sum (2) one can use relationships (5). After trivial transformations \(\sigma_{t}^{NN} \to \sigma_{t}^{A1N}, \quad B = B^{NN} \to B^{A1N}\), and neglected by the effects of 3 and 4 multiplicities screening the profile function can be presented as a sum of two terms [12,15]

\[
\omega_{1} = \sum_{i=1}^{n} \omega(b - \tilde{s} - s_{i})
\]

\[
\omega_{2} = \sum_{i \neq k} \omega(b - \tilde{s} - s_{i}) \cdot \omega(b - \tilde{s} - s_{k})
\]

According to (6) an amplitude \(F_{1}^{(1)}\) can be presented as difference of \(F_{1}^{(1)}\) and \(F_{2}^{(1)}\) amplitudes and in the case of \(\alpha A\) - scattering we have

\[
F_{1}^{(1)}(q, \tilde{s}) = - \frac{ip}{2\pi} K(q) \int db \cdot \exp i\tilde{q} \tilde{b} \cdot \omega_{1}(b - \tilde{s}, \{s\}) \cdot \exp \left[ - \int \omega(b - \tilde{s}, \{s\}) T_{A}(\tilde{s}) d^{2}\tilde{s} \right] \sum_{i=1}^{4} \rho_{\alpha}(r_{i}) dr_{i} \]

\[
F_{2}^{(1)}(q, \tilde{s}) = - \frac{ip}{2\pi} K(q) \int db \cdot \exp i\tilde{q} \tilde{b} \cdot \omega_{2}(b - \tilde{s}, \{s\}) \cdot \exp \left[ - \int \omega(b - \tilde{s}, \{s\}) T_{A}(\tilde{s}) d^{2}\tilde{s} \right]
\]

Let's remind, that in the approximation of rigid projectile nucleus an expression for cross-sections of all processes of light nucleus \(A_{1}\) scattering on heavy nucleus \(A_{2}\) has the form which coincides with expression for similar values of hadron – nuclear scattering processes but only with difference that hN-scattering amplitude therein should be replaced by \(A_{1}N\)-scattering amplitudes. Technically it means that the procedure of composite functions averaging for \(\omega_{A1N}(b - \tilde{s}, \{s\})\) operators over \(A_{1}\) coordinates is reduced to their replacement by functions from mean values of these operators, which can be coincide as profile functions of elastic \(A_{1}N\) scattering. In particular, total cross-section of quasielastic scattering in the approximation of hard \(\alpha\) - particle is given by expression

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \int d^{2}b_{1} d^{2}b_{2} \exp iq(b_{1} - b_{2}) \cdot \exp [ - \Phi(b_{1}) - \Phi^{*}(b_{2}) ] \cdot [\exp \Omega(b_{1} b_{2} - 1)]
\]
where
\[ \Phi(b) = \int \omega_{\alpha N}(\vec{b} - s)T_{\alpha}(s)d^2s \]
\[ \Omega(b_1,b_2) = \int \omega_{\alpha N}(\vec{b}_1 - \vec{s})\omega^*(\vec{b}_2 - s)T_{\alpha}(s)d^2s \]

If, as usual, the density distribution of nucleons in \( \alpha \)-particle is chosen to exponential, then the profile function \( \omega_{\alpha N} \) can be written as linear combinations of Gauss exponents. Partial cross-sections of n-multiplicities of quasielastic \( \alpha A \) scattering are obtained in [11,12].

Let's consider the process of \( A_1 \) and \( A_2 \) nuclei interaction in the result of which \( A_1 \) nucleus remains in its ground state and \( A_2 \) nucleus can be arbitrary excited. In this case it is convenient at first to proceed to optical limit on the nuclear to a number of conserved \( A_1 \) nuclei and to write the profile function of \( A_1 + A_2 \rightarrow A_1 + A_2^* \) process in the form [1,7,8]

\[ \omega_{A_1}(\{b\}) = \prod_{k=1}^A \omega_{A_1}(b_k, \{s\}) \]

The sum of \( A_1 \) nucleus scattering cross-sections, accompanying by possible excitations of nucleus \( A_2 \) with completeness condition for wave functions is given by expression

\[ \frac{d\sigma}{d\Omega} = \left(\frac{p}{2\pi}\right)^2 \int db_1 db_2 \exp iq(b_1 - \bar{b}_2) \prod_{k=1}^A \frac{T_{A_2}(\vec{s}_k)d^2\vec{s}_k}{A_2} \]

\[ \cdot \{1 - \exp[-\int \omega_{A_2 N}(s, \{s\})T_{A_1}(\vec{b} - \vec{s})d^2s] - \exp[-\int \omega_{A_2 N}(s, \{s\})T_{A_1}(\vec{b}_1 - \vec{s})d^2s] - \}

\[ - \exp[-\int \omega_{A_2 N}(s, \{s\})T_{A_1}(\vec{b}_2 - \vec{s})d^2s]\}

Condition for real values \( \omega_{A_2 N} = \omega_{A_2 N}^* \) of operator \( \omega_{A_2 N} \) is equivalent to the neglection of real part of NN- scattering amplitude in comparison with imaginary part that at high energies is a good approximation. By excluding from the total cross-section (11) the cross-section of elastic scattering, it is possible to obtain the cross-section of \( A_1 \) nucleus quasielastic scattering on target nucleus \( A_2 \) in optical limit of atomic numbers for both nuclei in the following form:

\[ \frac{d\sigma}{d\Omega} = \left(\frac{p}{2\pi}\right)^2 \int db_1 db_2 \exp iq(b_1 - \bar{b}_2) \cdot Q(b_1,b_2), \]

where

\[ Q(b_1,b_2) = \left\{ \begin{array}{ll}
\exp^{2\frac{\nu}{2}} & \left[ \frac{v}{2}(T_1(b_1 - s) + T_1(b_2 - s))T_2(s) \right]

\exp^{\frac{\nu}{2}} & \left[ \frac{v}{2}(T_1(b_1 - s), T_2(s) + f \left[ \frac{v}{2}T_1(b_2 - s), T_2(s) \right] \right]
\end{array} \right\} \]

From (12) one can see that of \( A_1 A_2 \) quasielastic scattering cross-section calculation in optical approximation over atomic numbers of colliding nuclei requires operations of five-time integration that, in general, is a simple problem at the presence of oscillating factor \( \exp iq(b_1 - b_2) \). However on the base of qualitative analysis carried out in [8-14], it is possible to restore cross-sections of quasielastic scatterings by integral characteristics of \( \sigma_{q,el} \) and \( <q^2> \) types, which are determined in [12]. As an example, the cross-section of quasielastic \( \alpha A \) - scattering can be calculated by using formula (12) and reduced values of \( \frac{d\sigma}{dt} \) at (0) and \( B_{\alpha A} \) parameters and adding to it a cross-section of elastic scattering, calculated in [12].
can obtain the total cross-section. It is possible to compare the calculated results with experimental data [2,8,11]. Results of such comparison are shown on the Fig.1 and Fig.2.

### Figure 1

Differential cross sections of $\alpha + ^{27}\text{Al}$ and $\alpha + ^{64}\text{Cu}$ scattering in optical approximation at $P_\alpha=17.9$ GeV/s in the region of transferred momentum $0 < q < 2$ fm$^{-1}$.

Solid curves I and II describes total (elastic and quazielastic) cross section. Lower curve correspond to the case, when both contributions from elastic and inelastic scattering are calculated in the framework of optical approximation. Upper curve correspond to the case, when contribution from elastic scattering was calculated in the frame of perturbation theory by shadow effects. Separately (dashed) is shown contribution from quazielastic scattering, calculated in the optical approximation [12,15]. For illustration of Coulomb contributions, on the same pictures is presented contribution from elastic scattering, calculated in optical approximation with taking into account coulomb effects (dashed – dotted curve) and without coulomb effects (dashed curve).

### REFERENCES


Section II. Basic problems of nuclear physics