ASYMPTOTIC NORMALIZATION COEFFICIENTS FOR MIRROR NUCLEI $^{27}$Al, $^{27}$Si AND NUCLEUS $^{28}$Si FROM ANALYSIS OF ONE NUCLEON TRANSFER REACTIONS

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INTRODUCTION

The experimental differential cross sections of one nucleon transfer reactions $^{27}$Al($d, l$)$^{26}$Al, $^{27}$Al($^3$He,$d$)$^{28}$Si and $^{28}$Si ($d^3$He)$^{27}$Al at beam energy of about several MeV/nucleon have been analyzed by method combining DWBA and Dispersion theory to obtain the values of vertex constants (VC) of single particle bindings $^{27}$Al$\rightarrow^{26}$Al+$n$, $^{28}$Si$\rightarrow^{27}$Al+$p$. In the frame of DWBA VC is defined by square asymptotic coefficient of bound state function $b_i^2$ multiplied by spectroscopic factor ($S$).

The shell model spectroscopic factors of proton and neutron bindings in mirror nuclei as well as spectroscopic factors of bindings of proton and neutron in nuclei with N=Z are supposed to be equal. So the relation between the VCs of corresponding states is to be defined by the relation of values of their $b_i^2$, which have been found in independent research on base of $pn$-pairs interaction approach with using Shr"{o}dinger equation formalism. Knowing the values of VC of
METHOD COMBINING DWBA AND DISPERSION THEORY

A long standing problem in using DWBA approach is that one has to use too many model parameters to obtain the values of spectroscopic factor of the single particle bound state from analysis of one nucleon transfer reactions, which make the obtained values not reliable. In the method called Combining Method (CM) [1,2] the problem was partly solved by introducing VC in the wave function approach. The notion of VC comes from dispersion theory of direct nuclear reactions. The VC has meaning of asymptotic normalization coefficient (ANC) of overlap function of A and B nuclei in reaction \( A(a,b)B \) and it is an important characteristic of bound state of nucleus. In frame of Schrödinger equation wave function approach the value of VC refers to \( S_{b,j}^{2} = \frac{\langle \psi_{b} | \psi_{j} \rangle}{\langle \psi_{b} | \psi_{j} \rangle} \).

The CM allows one to select among direct reactions pure peripheral ones with the domination of one-step mechanism of nucleon transferring (pole diagram in reaction amplitude). In such kind of a reaction the empirical values of \( S_{b,j}^{2} \) obtained in frame of classic DWBA is a constant value and depends on neither geometrical parameters of bound state potential used in the analysis nor the energy initiating the reaction. Therefore empirical values of VC’s obtained from analysis of experimental data by CM are free from uncertainty in bound state potential parameters used in calculations. The consideration of pure peripheral reactions is helping in diminishing uncertainty in choosing optical potentials to describe elastic channels too, because asymptotic part distorted waves are mostly defined by Coulomb potential. In this case different optical potentials used in calculation give deviation in outgoing values within 10-20%. Our previous work showed that the reactions \( (d,t) \) and \( (^{3}\text{He},d) \) are just of such kind and are appropriate for the analysis. For a set of 1p shell nuclei the values of VC have been obtained with error of about 10-20% [1,2].

OBTAINING VALUE \( b_{y} \)

A model based on an idea that proton-neutron pairs interactions are responsible for the nuclear structure is proposed recently [3]. In the approach it was established that proton and neutron of one pair have equal nuclear potentials. The principle of equivalence of belonging to one pair proton and neutron nuclear bound state potentials was used in obtaining Coulomb energy, surface tension energy from analysis of radii and binding energy of nuclei with \( N=Z \). The rules were spread for heavier nuclei to calculate charge radii with average deviation in 0.05 fm and binding energy with average deviation in 0.5% of the most abundant isotopes or nearest to them of all elements (only isotopes with even number of excess neutrons are considered) [3].

The bound states of belonging to the same pair proton and neutron were considered in Schrödinger equation formalism to obtain the value \( b_{y} \) [4]. The principle of equivalence of neutron and proton potentials (EPN) in mirror nuclei and in nuclei with \( N=Z \) has been used to obtain single particle bound state potential on values of root mean square radii of nuclei \( <r_{A}^{2}>^{1/2} \).

In such calculations root mean square radius of proton’s distribution \( <r_{p}^{2}>^{1/2} \) always slightly bigger than radius \( <r_{n}^{2}>^{1/2} \) of neutron’s one. Using the EPN principle makes Coulomb radius \( R_{c} \) a crucial parameter to fit radius of proton positioning \( <r_{p}^{2}>^{1/2} \) to get agreement between experimental value of radius of the nucleus \( <r_{A}^{2}>^{1/2}\exp \) and the value of root mean square radius of nucleus A \( <r_{A}^{2}>^{1/2} \). The values of \( <r_{A}^{2}>^{1/2} \) are calculated according to shell model \( <r_{A}^{2}>^{1/2} = (Z<r_{A}^{2}> + n_{y}<r_{p}^{2}>)/Z \), where \( Z' \) and \( A' \) denote nucleus with completed shell, \( n_{y} \) stands for number of protons above the shell. In the case \( Z' = 10 \), the geometrical parameters of nuclear potential, those are radius \( r_{o} \) and depth \( V_{o} \), are searched with fixed \( a = 0.55 \) Fm under the demand of well-depth procedure and EPN principle. Deviation of \( <r_{p}^{2}>^{1/2} \) in dependence on

\[ |\langle \psi_{b} | \psi_{j} \rangle|^{2} = \frac{S_{b,j}^{2}}{\langle \psi_{b} | \psi_{j} \rangle} \]
value of diffuseness varied within \( a = 0.50 \pm 0.65 \) Fm is less than 0.5% and deviation of values of \( b_j \) is within 3% [4].

**RESULTS OF ANALYSIS**

In the model of \( pn \)-pairs interactions for the nucleus \( ^{28}\text{Si} \rightarrow ^{27}\text{Al} + p \) the values \( <r_p^2> = 3.58 \) Fm and \( <r_A^2> = 3.13 \) Fm were found in [3]. In Shrödinger formalism of using experimental single particle binding energy to describe potential of Woods–Saxon form taking into account spin-orbital interaction under the condition of EPN principle the value \( <r_p^2> \) has been fitted with \( R_c = 3.300 \) Fm. The values of parameters of the potential together with the values of \( <r_n^2> \) and \( b_j \) of neutron binding in \( ^{28}\text{Si} \rightarrow ^{27}\text{Si} + n \) are given in the Table below.

**Table.**

| Nucleus A | \( V_0 \) [MeV], \( r_0 \) [Fm], \( a \) [Fm] | \( b_{ij} \) [Fm] | \( <r_{p/n}^2>^{1/2} \) [Fm] | \( <r_A^2>^{1/2} \) [Fm] | \( |G^2| \) [Fm] | \( C^2S \) |
|-----------|----------------------|--------|-----------------|-----------------|-------------|--------|
| \( ^{27}\text{Al} \rightarrow ^{26}\text{Al} + p \) | -47.405, 1.355, 0.55 | 7.44 | 3.50 | 3.144 | 6.54 [7]; 5.65±7.33 [5,6] | 0.79; 0.68±0.89 |
| \( ^{27}\text{Si} \rightarrow ^{26}\text{Al} + p \) | 12.2 | 3.60 | 150., 155., 173., 175., 179. |
| \( ^{28}\text{Si} \rightarrow ^{27}\text{Si} + n \) | -50.381, 1.393, 0.55 | 14.3 | 3.49 | 56.0 [11], 57.8 [10], 64.5 [12], 65.3 [8], 66.8 [9] | 1.8 ± 2.2 |
| \( ^{28}\text{Si} \rightarrow ^{27}\text{Al} + p \) | 23.4 | 3.58 [3] | 150., 155., 173., 175., 179. |

Using the same value of Coulomb radius \( R_c \) the values of \( <r_{p/n}^2> \) for bindings \( ^{27}\text{Si} \rightarrow ^{26}\text{Al} + p \) and \( ^{27}\text{Al} \rightarrow ^{26}\text{Al} + n \) were found under the condition of EPN. The obtained values together with parameters of the potential are given in the Table.

Some experimental data on measurement of differential cross sections of the reactions \( ^{27}\text{Al}(d,t)^{26}\text{Al} \) [5-7], \( ^{27}\text{Al}(^3\text{He},d)^{28}\text{Si} \) [8, 9] and \( ^{28}\text{Si}(d^3\text{He})^{27}\text{Al} \) [10-12] at the beam energy of about several MeV/nucleon have been analyzed to obtain values of VC \( |G^2| \) by the CM method. Taking into account the fact that the spectroscopic factors of single particle binding states of mirror nuclei are supposed to be equal one can obtain the values of missed \( |G^2| \) from equation

\[
|G^2|_{27\text{Al}}/|G^2|_{27\text{Si}} = b_{ij}^2 {^{27}\text{Al}}/b_{ij}^2 {^{27}\text{Si}}
\]

The same way is used for proton and neutron bound states in nucleus \( ^{28}\text{Si} \). The obtained values of VC are given in the Table in accordance with the values of VC estimated by the CM method on data from the works given in references. The empirical values of \( S \) obtained in the work are given in the Table in accordance with the values of VC’s.

From the Table one can see that the difference in the values of VC is within 20 %, which is defined by the experimental errors of measurement and uncertainty in choosing parameters of optical potentials. So all obtained values in the works are supposed to have the same reliability. For the spectroscopic factors the 3% of uncertainty of used diffuseness \( a = 0.55 \) Fm should be taken into account too.

**CONCLUSION**

Vertex constants of \( ^{27}\text{Al} \rightarrow ^{26}\text{Al} + n \) and \( ^{28}\text{Si} \rightarrow ^{27}\text{Al} + p \) have been found by means of CM method providing a reliable value of VC with errors estimated within 20% defined by measurement errors and uncertainty in optical potentials used. The values of \( b_{ij} \) of bound states \( ^{27}\text{Al} \rightarrow ^{26}\text{Al} + n, ^{28}\text{Si} \rightarrow ^{27}\text{Al} + p \) and \( ^{27}\text{Si} \rightarrow ^{26}\text{Al} + p, ^{28}\text{Si} \rightarrow ^{27}\text{Si} + n \) have been obtained in approach of \( pn \)-pairs interactions model, where the principle of equivalence of proton and neutron nuclear potential is used. Then the values of VC of bound states of \( ^{27}\text{Si} \rightarrow ^{26}\text{Al} + p \) and \( ^{28}\text{Si} \rightarrow ^{27}\text{Si} + n \) and
spectroscopic factors of all states under consideration were obtained with error expected to be about 20%.

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REFERENCES