QUADRUPOLE DEFORMATIONS OF FISSION FRAGMENTS IN SCISSION POINT

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ABSTRACT
In the presented paper we discuss the possibility to extract the magnitude of quadrupole deformations for asymmetric fission fragments in the scission point from experimental data about their total kinetic energies and excitation energies of fission fragments and target nucleus. We start from the simple case of two coaxial ellipsoids of revolution with uniformly distributed charge and nuclear matter density. Influence of nuclear forces between fission fragments in the scission point was neglected. Shell model corrections and contribution from nuclear part of interaction in scission point are discussed. Separate attention was paid to the connection between total kinetic energies of fission fragments and their time of flight from scission point up to point of their detection.

MOTIVATION
The whole picture of the nuclear fission can be roughly divided into the three important stages:
- First -- from initial state of target nucleus and projectile particle (like γ quanta, thermal neutron and etc.) up to the classical threshold of fission - saddle point.
- Second -- from the saddle point up to the scission point.
- Third -- from the scission point up to the detection of fission fragments.
From this simple consideration we can conclude, that last stage of fission process is a reflection of fission dynamic and our aim is to extract all possible information about shapes of fission products from their total kinetic energies (TKE), excitation energies, and mass and charge distributions.

MODEL ASSUMPTIONS
Our assumptions are:
- Both fission fragments have uniform distribution of mass and charge
- They have simple configuration consisting from two coaxial ellipsoids of revolution with axis of symmetry along their line of fission.
GENERAL APPROACH AND NUMERICAL RESULTS

Introducing notations: $E_1, E_2$ - kinetic energies in the c.m., $Z_1, Z_2$ - electric charges and $A_1, A_2$ are mass numbers of fission fragments, $a_1, c_1, a_2, c_2$ are their corresponding semiaxes. In the first approximation, if we neglect the influence of nuclear forces between fission fragments, we can write from conservation law following expression [1]:

$$ E_1 \left( 1 + \frac{A_1}{A_2} \right) = \frac{e^2 Z_1 Z_2}{D_0} F(x, y) $$

(1)

where $D_0 = d_0 + c_1 + c_2$ and $x$, $y$ and $F(x, y)$ are defined as in [1]:

$$ F(x, y) \approx 0.75 \left[ \frac{1}{x} \right] \ln \frac{1+x}{1-x} + \left[ \frac{1}{y} \right] \ln \frac{1+y}{1-y} + 1.5 \left( \frac{1}{x^2} + \frac{1}{y^2} \right) + 0.24 + 0.257(x^2 + y^2) + 0.26(x^4 + y^4) + 0.514 x^2 y^2 x^2 y^2 - 1 $$

(2)

$$(x = \frac{c_1}{D_0} \left[ 1 - \left( \frac{a_1}{c_1} \right)^2 \right], a_1 < c_1 \text{ and } x = \frac{c_2}{D_0} \left[ 1 - \left( \frac{a_2}{c_2} \right)^2 \right], a_2 < c_2.)$$

Introducing new parameter of ratio $k = \frac{a_1}{a_2}$ finally we have:

$$ E_1 \left( 1 + \frac{A_1}{A_2} \right) = \frac{a_1^2 \alpha \hbar c Z_1 Z_2}{r_0^3 (A_1 + A_2 k^2) + a_1^2 d_0} F(x, y) $$

(3)

where $\alpha$ is fine structure constant, $c$ - speed of light and $r_0 = \frac{\hbar}{m_c}$. This last equation (3) can be solved numerically relatively variable $a_1$ and after that we can determine all values, including $c_1$, $a_2$, $c_2$, eccentricities and quadrupole deformations of fission fragments. After realization of numerical procedure for two different set of input parameters: $d_0 = 0.0$ (0.5), $k = 0.5$ (0.8) at $E_1 = 90$ MeV and some probe values $Z_i = 48, A_i = 100, Z_i = 50, A_i = 152$ we have:

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$c_1$</th>
<th>$\varepsilon_1$</th>
<th>$\beta_1$</th>
<th>$a_2$</th>
<th>$c_2$</th>
<th>$\varepsilon_2$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.93</td>
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<td>0.98</td>
<td>6.81</td>
<td>7.87</td>
<td>6.88</td>
<td>0.9</td>
<td>-0.42</td>
</tr>
<tr>
<td>4.83</td>
<td>12.0</td>
<td>0.91</td>
<td>3.47</td>
<td>6.04</td>
<td>11.67</td>
<td>0.86</td>
<td>2.372</td>
</tr>
</tbody>
</table>

At the end we want to underline that in this approach it is possible to obtain different results for the one set of input parameters, which means that above obtained procedure is ambiguous. To avoid it we must to use additional information about excitation energies of fission fragments. In this case, for spontaneous fission (s.f) it is well established that prescission kinetic energy of fission fragments is close to zero [2,3] and we can directly present the excitation energy at s.f. as:

$$ \Delta E_{exc} = [M_0 - (M_1 + M_2)]c^2 - E_0 $$

where $M_0$ is the mass of initial nucleus, $M_1, M_2$ are masses of fission fragments and $E_0$ is their TKE. Using [4], we can write that $\Delta E_{exc} = \Delta E_{exc1} + \Delta E_{exc2}$, where, by neglecting proximity forces in scission point we have:
\[ \Delta E_{\text{excl}} = 4\pi \epsilon_1^{2/3} \sigma_1 \begin{bmatrix} A\tan \left( \frac{\epsilon_1}{\sqrt{1-\epsilon_1^2}} \right) \end{bmatrix} - 3e^2 Z_1^2 \begin{bmatrix} \frac{(1-\epsilon_1^2)^{1/3}}{2\epsilon_1} \ln \frac{1+\epsilon_1}{1-\epsilon_1} - 1 \end{bmatrix} \frac{5r_0\epsilon_1^{1/3}}{A_1^{1/3}} \]

\[ \Delta E_{\text{exc}} = 4\pi \epsilon_2^{2/3} \sigma_2 \begin{bmatrix} A\tan \left( \frac{\epsilon_2}{\sqrt{1-\epsilon_2^2}} \right) \end{bmatrix} - 3e^2 Z_2^2 \begin{bmatrix} \frac{(1-\epsilon_2^2)^{1/3}}{2\epsilon_2} \ln \frac{1+\epsilon_2}{1-\epsilon_2} - 1 \end{bmatrix} \frac{5r_0\epsilon_2^{1/3}}{A_2^{1/3}} \]

Here \( \sigma_1 \) and \( \sigma_2 \) are surface tensions of fission fragments and \( \epsilon_1, \epsilon_2 \) are their eccentricities:

\[ \epsilon_1 = \sqrt{1 - \frac{1}{A_1^2} \left( \frac{a_1}{r_0} \right)^6} \quad \text{and} \quad \epsilon_2 = \sqrt{1 - \frac{1}{A_2^2} \left( \frac{a_2}{c_2} \right)^6} \]

Using simple modification of (3) we can simply to write for determination of \( a_1 \) and \( a_2 \) values following set of algebraic equations:

\[ E_i \left( 1 + \frac{A_1}{A_2} \right) = \frac{a_i^2 \alpha \gamma c Z_i^2}{r_0} \begin{bmatrix} A_1 + A_2 \left( \frac{a_1}{a_2} \right)^2 \end{bmatrix} + a_i^3 d_0 \]

\[ [M_0 - (M_1 + M_2)]c^2 - E_0 = \Delta E_{\text{excl}} + \Delta E_{\text{exc2}} \]

As a numerical example we can calculate parameters of shape for s.f. of \( ^{252}\text{Cf} \rightarrow ^{104}\text{Mo} + ^{148}\text{Ba} \) reaction for two sets of initial parameters at \( E_i = 90 \text{ MeV} \) and \( E_i = 50 \text{ MeV} \). In both cases we take \( d_0 = 0.5 \text{ fm} \).

**Table 2**

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( c_1 )</th>
<th>( \epsilon_1 )</th>
<th>( \beta_1 )</th>
<th>( a_2 )</th>
<th>( c_2 )</th>
<th>( \epsilon_2 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6</td>
<td>6.7</td>
<td>0.17</td>
<td>0.046</td>
<td>7.49</td>
<td>7.4</td>
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<td>-0.036</td>
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<td>4.49</td>
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<td>0.95</td>
<td>4.765</td>
<td>12.96</td>
<td>2.47</td>
<td>0.98</td>
<td>-4.46</td>
</tr>
</tbody>
</table>

**CONCLUSION**

From the above obtained results one can see that the values of quadrupole deformations are more preferable at energy of detected fission fragment of \( E_i = 90 \text{ MeV} \). Physically it is obvious, because in this case the energy is distributed between their TKE and fission fragments deformation energy. Iteration procedure for the solution set of algebraic equations (6),(7) is numerically stable, and in the case at \( E_i = 90 \text{ MeV} \) we have rather good convergence. These investigations are not yet complete and above obtained results are preliminary.

**REFERENCES**