ABSTRACT:
Natural circulation is an important passive heat removal mechanism for both existing and next generation light water reactors. Simplified Boiling Water Reactor (SBWR) is one of the advanced light water reactors that rely on natural circulation for normal as well as emergency core cooling. In this study, basic natural circulation characteristics of this reactor are examined on a flow loop that simulates the operation of SBWR. On this model, effect of system operating parameters on the steady state natural circulation characteristics inside the loop is studied via solving the transcendental equation for loop flow rate.

INTRODUCTION:
Natural circulation is an important heat removal mechanism in advanced light water reactor technology. It has also many other industrial applications owing to its simplicity and passivity. The basic steady-state natural circulation characteristics of SBWR, based on flow loop shown in Figure 1, are studied.

Flow in natural circulation or gravity-driven systems is induced by density differential between inside and outside of the heater-riser section. Density differential is caused by heat addition in the heater part of the loop. Therefore, heater or core power level and determination of void distribution inside the core section are important issues in the analysis of natural circulation loops.

Figure 1: Natural Circulation Loop Model for SBWR.

Core power is the most important parameter in natural circulation systems since it drives the flow via creating the density differential. The relationship between the loop flow rate and the heater power (core power) is the key parameter when analyzing the steady state and transient characteristics of the natural circulation flow inside the loop. In single-phase flow, or in single-
phase natural circulation loops, it can be shown analytically that loop flow rate is proportional to $1/3$ power of the heater power [1].

However, in two-phase natural circulation systems, loop flow rate is not a simple function of heater power as in single-phase flow loops. This is because of non-monotonic change in the gravitational and frictional pressure drops with heating power.

It has been shown in this article that under some operating conditions, loop flow rate vs. heater power curve is multi-valued. In other words, under certain circumstances, one can find more than one steady-state solution for given heater power. This may lead the loop flow rate to bifurcate for certain value of heater power and cause steep reduction of flow rate on the loop flow rate vs. heater power curve. This phenomenon is called Static Bifurcation [2]. This is important phenomena and should be studied carefully, since multiple steady state solutions cannot be observed in experiments.

In this study, first, transcendental equation for loop flow rate is derived for the loop in Figure 1. Then, the loop flow rate vs. heater power curve is obtained by solving this equation. Based on this curve, static bifurcation is investigated and the effects of system parameters on bifurcation are studied with details. For two-phase flow modeling, Drift Flux Flow Model is used and the results are compared with Homogenous Equilibrium Model (HEM).

**Model Description:**

The flow loop in Figure 1 is a representative picture of a natural circulation boiling water reactor. This model is flexible and can be easily adapted to any other systems in which flow is induced by gravity.

As shown in Figure 1, flow loop is composed of mainly three parts, namely:

- **Heater Section**, which represents the SBWR fuel bundle. Two types of axial power profile are considered;
  1. Constant Power Profile, $f(z) = 1$
  2. Bottom Peak Profile, $f(z) = \pi(H - z) / H \sin\left(\frac{\pi(H - z)}{H}\right)$

- **Riser Section**, which is the chimney section above the core. It is used to enhance the flow via increasing the gravity head. It also increases the mass inventory of the reactor vessel.

- **Condenser** is the part in which outlet temperature of the fluid is kept constant.

- **Downcomer Section** is the part in which liquid flows downward and gravity head that is necessary for flow increased.
Transcendental Equation For Loop Flow Rate:

Transcendental equation for loop flow rate is derived by solving the appropriate forms of the governing equation for single and two-phase flow regions in the heater part. These equations and the solutions are listed in the following table:

<table>
<thead>
<tr>
<th>DEFINITION:</th>
<th>EQUATION:</th>
<th>SOLUTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Phase</td>
<td>Continuity</td>
<td>$\frac{dW}{dz} = 0$</td>
</tr>
<tr>
<td>Single Phase</td>
<td>Momentum</td>
<td>$\frac{dP}{dz} = f \frac{W^2}{2A_{in} \rho(z)} + \rho(z)g$</td>
</tr>
<tr>
<td>Energy Equation</td>
<td></td>
<td>$\frac{dh}{dz} = -\frac{h_{fg}}{\rho_m U_m}$</td>
</tr>
<tr>
<td>Two Phase</td>
<td>Mixture Continuity</td>
<td>$\frac{dj}{dz} = \Gamma_{sat}(z) V_{fj}$</td>
</tr>
<tr>
<td>Two Phase</td>
<td>Vapor Continuity</td>
<td>$\frac{d(\alpha U_g)}{dz} = \Gamma_{sat}(z) V_{kg}$</td>
</tr>
<tr>
<td>Two Phase</td>
<td>Momentum Equation</td>
<td>$\frac{d\rho}{dz} = \rho_m \frac{\Delta \rho}{2 \Delta z} V_f + \left[ \frac{\Delta \rho}{\Delta z} \frac{\alpha}{\rho} \right] V_f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ \frac{C_{ij}}{2} \left[ \frac{\Delta \rho}{\Delta z} \frac{\alpha}{\rho} \right] V_{ij} + K_n \frac{C_{ij}}{2} \Omega_{ij}$</td>
</tr>
</tbody>
</table>

$C_0$ is the distribution parameter and $U_{fg}$ is the drift velocity in Drift Flux Model[3]. $z_s$ is the point at which saturated boiling starts. In this table, the solution of the two phase momentum equation is written by using HEM. For Drift Flux Model, the solution cannot be written in analytical form, rather it is written in integral forms.
The list of basic simplifying assumptions in the analysis is as follows;

- Subcooled boiling is neglected.
- Inlet subcooling is independent parameter and given externally. This assumption is based on that condenser has enough condensation capability for keeping the outlet temperature at a constant value.
- Riser and downcomer sections are adiabatic, that is there is no heat exchange with surroundings. Furthermore, there is no frictional loss inside the downcomer.
- Linear liquid density variation is assumed in the single-phase flow region inside the heater.

If the frictional loss is neglected in the downcomer section, gravitational head provided in this part is written as \( \rho_{in}gH_{dc} \). This head should balance the all pressure loss throughout the loop, therefore transcendental equation for loop flow rate is then written as,

\[
\rho_{in}gH_{dc} = (\Delta P)_{1phi} + (\Delta P)_{2phi} + (\Delta P)_R = \sum_{loop} (\Delta P)
\]

This equation can be rewritten as,

\[
f(U_{in}', \{q\}') = \rho_{in}gH_{dc} - \sum_{loop} (\Delta P) = 0
\]

It is the transcendental equation for loop flow rate from which loop flow rate vs. heater power curve is plotted.

**Solution Method:**

In this study, expression for \( f(U_{in}', \{q\}') = 0 \) is solved for loop flow rate for given heater powers. The solution is performed via Bisection Method. Let’s consider a fixed heater power value. If the function \( f \) changes sign in the velocity interval \([U_{in}^*, U_{in}^* + \Delta u]\), then there exists at least one steady-state solution that satisfies the balance described before. Then, one can calculate the root via interpolation, and loop flow rate is determined as

\[
w_{loop} = U_{in}^* A_{it} \rho_{in} - \frac{\Delta uf(U_{in}^*, \{q\}')} {f(U_{in}^* + \Delta u, \{q\}') - f(U_{in}^*, \{q\}')}
\]

If the interval \( \Delta u \) is chosen sufficiently small, then the accuracy of the result is high and multiple solutions are determined and resolved quite well. By means of the solution of this equation, following curve, which is also called Bifurcation Diagram, is obtained. The figure implies that there may exist more than one steady state solution for given bundle power. It is the case especially at low system pressures.
Two types of flow model are adopted to analyze the effect of flow model on the steady-state characteristics of natural circulation loops, namely HEM and DRIFT FLUX MODELING. The result is shown in the following figure.

The effects of axial power shape, system pressure, inlet subcooling and local flow resistances on the bifurcation characteristics of the natural circulation loop are studied. Results are given in the following figures:

**Figure 2. Typical Bifurcation Diagram**

**Figure 3. Effect of Axial Power Shape**

**Figure 4. Effect of System Pressure**

**Figure 5. Effect of Inlet Subcooling**

**Figure 6. Effect of Inlet Loss Coefficient**
CONCLUSION:
In this study, basic natural circulation characteristics of SBWR were investigated based on the loop shown in Figure 1. By using the transcendental equation for loop flow rate, steady-state solutions for the loop were determined for different bundle powers and loop flow rate vs. bundle power curves were obtained. Based on this curve, static bifurcation that may occur under some operating conditions was examined. Some operating parameters such as system pressure, inlet subcooling, and local flow resistances have been evaluated to reveal their effects on the performance of such natural circulation loops.

In this article, it has been shown that under some operating conditions, especially at low pressures, multiple steady-state solutions exist in such systems. Bifurcation phenomenon is pronounced in natural circulation loops in some cases and it causes the loop flow rate to drop drastically after certain value of bundle power. During the analysis, role of the operating parameters on this phenomenon was investigated. It has been prove that increasing the system pressure causes the bifurcation point to vanish and stabilizes the loop. The other conclusions are summarized as follows;

- Increasing the inlet subcooling broadens the region of multiple steady-state solutions. Therefore, decreasing the inlet subcooling can eliminate static bifurcation.

- Effect of local flow resistances on the performance of the natural circulation loop depends on the location of the restriction. When it is applied at the bundle inlet, increasing the value of loss coefficient eliminates multiple-solution region. However, it is applied at the bundle exit, bifurcation point cannot be eliminated, moreover, maximum heat load of the natural circulation loop decreases by increasing the exit lost coefficient.

- Two-phase flow model has great importance since the general shape of the loop flow rate vs. heater power curve changes significantly. For instance, the presence of slip between the phases causes the region of multiple solutions to get smaller.

- The effect of axial power shape on the performance of the loop was also considered. It was demonstrated that change in the power shape does not change the general for of the curve significantly. However, bottom peaked axial power shape causes the starting point of the multiple solution region to shift toward to lower power levels a little bit.
**Nomenclature:**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>P</td>
<td>Perimeter</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
</tr>
<tr>
<td>G</td>
<td>Mass Flux</td>
</tr>
<tr>
<td>H</td>
<td>Height</td>
</tr>
<tr>
<td>K</td>
<td>Loss Coefficient</td>
</tr>
<tr>
<td>W</td>
<td>Loop Flow Rate</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
</tr>
<tr>
<td>f</td>
<td>Friction factor</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational Acceleration</td>
</tr>
<tr>
<td>x</td>
<td>Flow Quality</td>
</tr>
<tr>
<td>z</td>
<td>Axial Coordinate</td>
</tr>
</tbody>
</table>

**Subscript:**

- sat: Saturated
- f: Liquid / Frictional
- g: Vapor / Gravitational
- H: Heater
- R: Riser
- dc: Downcomer
- lo: Liquid only
- m: Mixture
- in: Inlet
- ex: Exit
- e: Equivalent

**<->**: Averaging Operator

**Greek Letters:**

- \( \alpha \): Void Fraction
- \( \Gamma \): Volumetric Vapor Generation Rate
- \( \Phi \): Two Phase Friction Multiplier
- \( \nu \): Specific Volume
- \( \rho \): Density

**REFERENCES:**


2. Wei Tao, Bo Kuang and Jijun Xu, Thermal Siphon Instability Mechanism In a Two Phase Natural Circulation Systems, Ninth International Conference on Nuclear Reactor Thermal Hydraulics, October 3-8 1999